ALLOCATION AND REDUCTION OF RISK REQUIRED CAPITAL AFTER COMBINING OF THE UNITS

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Abstract

This paper addresses the effect of total capital by the combining allocation in multi-line financial businesses. General results are derived in the case of multivariate Normal risks. The key result of this paper is the reduction of capital required to each risk after occurrence of combination for all risks and how to allocate that capital to risks. The allocation methodology results can be applied to financial units.

1. **Introduction**

The subject of the determination of risk capital has been of active interest to researchers, of interest to regulators of financial institutions, and of direct interest to commercial vendors of financial products and services.
The confidence level chosen is arbitrary. In practice, it can be a high number such as 99.95% for the entire enterprise, or it can be much lower, such as 95% or 90%, for a single unit within the enterprise. This lower percentage may reflect the inter-unit diversification that exists.

The concept of Value-at-Risk (VaR) has become the standard risk measure used to evaluate exposure to risk. In general terms, the VaR is the amount of capital required to ensure, with a high degree of certainty, that the enterprise doesn't become technically insolvent. The degree of certainty chosen is arbitrary. In practice, it can be a high number such as 99.95% for the entire enterprise, or it can be much lower, such as 95%, for a single unit within the enterprise. This lower percentage may reflect the inter-unit diversification that exists.

The promotion of concepts such as VaR has prompted the study of risk measures by several authors (e.g. Wang, 1996, 1997). Specific desirable properties of risk measures were proposed as axioms in connection with risk pricing by Wang, Young and Panjer (1997) and more generally in risk measurement by Artzner (1999).

In this paper, we consider a random variable X_j representing the negative of the possible profits, i.e. the possible losses, arising from a business unit identified with subscript j. Then the total or aggregate losses for n units combined is simply the sum of the losses for all units

$$X = X_1 + X_2 + ...X_{n-1} + X_n$$

The probability distribution of the aggregate losses depends not only on the distributions of the losses for the individual units but also on the inter-relationships between them. Correlation is one such measure of inter-relationship. Correlation is, however, a simple linear relationship that may not capture many aspects of the relationship between the variables. However, it does perform perfectly for describing inter-relationships. Although the Normal assumption is used extensively in connection with the modeling of changes in the logarithm of prices in the stock market, it may not be entirely appropriate for modeling many processes including insurance loss processes.

2. Risk Measures

A *risk measure* is a mapping from the random variables representing the risks to the real line. A risk measure gives a single number that quantifies the risk exposure in a way that is meaningful for the problem at hand. The standard deviation of a distribution is a measure of risk. One of the other most commonly used risk measures in the fields of finance and statistics is the *quantile* or *Value-at-Risk.* This risk measure is the size of loss for which there is a small probability of exceedence. The following properties give the algebra of such measure:

1. Subadditivity:

$$P(X+Y) \le P(X) + P(Y)$$

This means that the capital requirement for two combined risks will not be greater than the sum of the capital requirements for the risks treated separately. This is necessary, since otherwise companies would have an advantage to disaggregate into smaller companies.

2. Monotonicity:

If $X \le Y$ for all possible outcomes, then $P(X) \le P(Y)$ This means that if the losses of one risk are smaller than those of another risk , then the capital requirement of the first is smaller than that of the second.

3. Positive Homogeneity:

For any positive constant λ , $P(\lambda X) = \lambda P(X)$ This means that the capital requirement is independent of the currency in which the risk is measured.

4. Translation invariance

For any positive constant α , $P(X+\alpha) = P(X) + \alpha$. This means that there is no additional capital requirement 'for an additional risk for which there is no uncertainty. In particular, by making X identically zero, the total capital required for a certain outcome is exactly the value of that outcome. Risk measures satisfying these criteria are deemed to be coherent. There are many such risk measures.

3. The *q*-quantile or VaR

The *q*-quantile , x_q , is the smallest value satisfying $\Pr\{X > x_q\} = 1 - q$.

As a risk measure, x_q is the Value-at-Risk and is used extensively in financial risk management of trading risk over a fixed time period.

The conditional tail expectation or TailVaR

The conditional tail expectation is given by

$$E[X | X > x_q]$$

This is called conditional tail expectation by Wirch (1997) and TailVaR by Artzner (1999). It can be seen that this will be larger that the VaR measure for the same vale of q described above since it is the VaR x_q plus the expected excess loss; i.e.,

$$E[X | X > x_q] = x_q + E[X - x_q | X > x_q].$$

Overbeck (2000) also discusses VaR and TailVaR as risk measures. TailVaR the provides the expected excess loss over that threshold, when the threshold has been exceeded. One can define the threshold x_q as $\rho(X) = E[X | X > x_q]$.

4. Allocation of Capital

Harry H. Pnjer (2002) discusses details of allocation total capital to combined risk units. Consider now that the random variable X and the allocation of capital to the individual risks $X_1, X_2, ..., X_n$ when the capital requirement $P(X)$ has been determined for the total risk X . Denault (2001) address this problem by defining a set of desirable properties for an allocation methodology. He defines a coherent allocation method as one that possesses those properties.

Let $K = P(X)$ represent the risk measure for the total risk X .
Let X_j denote the allocation of K to the i-th risk. The properties are:

1. Full allocation

$$K_1 + K_2 + ...K_{n-1} + K_n = K$$

This means that all of the capital is allocated to the risks.

2. No undercut

$$K_a + K_b + \ldots\ldots + K_z \leq P(X_a + X_b + \ldots\ldots + X_z)$$

for any subset {a, b, ... , z} of {1, 2, ... , n}.
This means that any decomposition of the total risk will not increase the capital from its value if the risks stood alone.

3. Symmetry

Within any decomposition, substitution of one risk X_i with an otherwise identical risk X_j will result in no change in the allocations.

4. Riskless allocation

The capital allocation (in excess of the mean)to a risk that has no uncertainty is zero. These properties seem to be reasonable and intuitive requirements for an allocation method. They are, however, not sufficient to characterize a single allocation method.

5. Important Notes on Bivariate Normal Risks

The Normal distribution is used extensively in financial applications. In this section, we use the Normal distribution to model the distribution of the present value of losses for a risk. The risk could be an entire company, such as an insurance company or other financial institution, or it could be a much smaller unit such as a block of insurance policies.

Consider the aggregate risk $X = X_1 + X_2 + \ldots. + X_n$
where the X_j s forms a multivariate Normal distribution. Note that X itself follows Normal distribution. Denoting its mean and variance by μ and σ^2, it is straightforward to show that the TailVaR can be written as

$$K = E(X / X > x_q) = \mu + \alpha\sigma^2$$

where $$\alpha = \frac{f(x_q)}{1 - F(x_q)}$$

and f and F are the probability density function and the corresponding cumulative distribution function of the Normal

distribution with mean μ and standard deviation σ.
To consider the individual allocations, it is sufficient to consider only the case with n = 2 by isolating one random variable (say X_1) and combining all the risks, except X_1, into the random variable X_2. This will simplify the notation considerably. So consider the aggregate risk

$$X = X_1 + X_2$$

In this case, with a bit of calculation, one finds the allocation to risk1

$$K_1 = E(X_1 / X > x_q) = \mu_1 + \alpha\sigma_1^{\;2}(1 + \rho_{1,2}\frac{\sigma_2}{\sigma_1})$$

where $\rho_{1,2}$ represents the correlation coefficient between X_1 and X_2. For the bivariate Normal model considered here, the size of the TailVaR for the total risk is, of course, dependent on the correlation coefficient.

If the two risks are uncorrelated, the capital allocation for the each risk is of the same form as the TailVaR for each if the risks taken separately on a stand-alone basis except that the factor α is based on the distribution of the sum of the two risks.

When the correlation coefficient is not equal to 1 the total capital to each risk after combination is less than the total capital to each risk before combination, see *Table A* cases from 1 to 15, except case 3 at which the correlation coefficient is equal to 1 . Therefore the total capital to each risk after combination is equal the total capital to each risk before combination also for cases from 16 to 19.

If the two risks are identical, the proportion allocated to each risk is always 50% of the total allocation independent of the correlation, see *Table A* cases1,2,3,4,6,7,9,13 and16.
If the correlation coefficient is negative and satisfies $\rho_{1,2} < -\frac{\sigma_1}{\sigma_2}$

, then the total capital allocated to risk 1 is less than the mean, see *Table A* cases 5,10,14,20,21,22 and 23.

When the correlation coefficient is negative and satisfies $\rho_{1,2} = -\frac{\sigma_1}{\sigma_2}$

, the total capital allocated to risk 1 is equal to the mean, see *Table A* cases 11 and 12.

The following table illustrates the allocation of capital with correspondence probabilities for the same 23 cases as in *Table A.*

Case	ρ	K	K1	K2	p(K)	p(K1)	p(K2)
1	0	15.5384	7.76918	7.76918	0.996	0.970	0.970
2	0.5	17.8489	8.92443	8.92443	0.996	0.990	0.990
3	1	32.6521	16.3261	16.3261	0.996	0.996	0.996
4	-0.5	1.33261	0.6663	0.6663	0.996	0.909	0.909
5	-1	15.3304	-0.33043	15.6609	0.996	0.500	0.996
6	0.5	6.15466	3.07733	3.07733	0.996	0.990	0.990
7	0.5	43.0814	21.5407	21.5407	0.996	0.990	0.990
8	0.5	54.103	24.0294	30.0736	0.996	0.978	0.994
9	-0.5	18.6652	9.33261	9.33261	0.996	0.909	0.909
10	-0.5	27.6096	8.2608	19.3488	0.996	0.230	0.995
11	-0.5	31.2326	11	20.2326	0.996	0.500	0.990
12	-0.25	26.0812	2.71828	23.3629	0.996	0.500	0.995
13	0.25	64.2141	32.107	32.107	0.996	0.982	0.982
14	-0.75	13.3075	-1.82689	15.1344	0.996	0.500	0.991
15	0.75	17.1955	7.55574	9.63974	0.996	0.987	0.996
16	1	18.6609	9.33043	9.33043	0.996	0.996	0.996
17	1	22.6565	9.99564	12.6609	0.996	0.996	0.996
18	1	31.6521	16.3261	15.3261	0.996	0.996	0.996
19	1	37.6478	16.9913	20.6565	0.996	0.996	0.996
20	-0.75	20.3075	1.17311	19.1344	0.996	0.173	0.991
21	-0.75	14.3075	-0.82689	15.1344	0.996	0.500	0.991
22	-1	14.6652	3.33479	11.3304	0.996	0.004	0.996
23	-0.9	12.1714	0.43527	11.7361	0.996	0.397	0.918

6. Allocation in the Multivariate Normal

Assume that there are two risks, then , the allocation formula for the first risk is

$$K_1 = \mu_1 + \alpha \sigma_1{}^2 (1 + \rho_{1,2} \frac{\sigma_2}{\sigma_1})$$

Where:

μ_1 : mean of the first risk

σ_1 : Standard deviation of the first risk

$\rho_{1,2}$: The correlation coefficient between the two risks

σ_2 : Standard deviation of the second risk

$$\alpha = \frac{f(x_q)}{1 - F(x_q)}$$

and f and F are the probability density function and the cumulative distribution function of the Normal distribution with mean μ and standard deviation σ.

Assume that there are *n* risks, the subscript j refers to the j th while negative – j refers to all but the j th risk. So that

$$x_{-j} = x_1 + x_2 + ... + x_{j-1} + x_{j+1} + ... + x_n$$

by replacing subscript 1 by j and subscript 2 by – j in allocation formula then

$$K_j = E(X_j / X > x_q) = \mu_j + \alpha \sigma_j{}^2 (1 + \rho_{j,-j} \frac{\sigma_{-j}}{\sigma_j})$$

note that

$$X = X_1 + X_2 + ...X_{n-1} + X_n$$

Since Covariance

$$(X_j, X) = \sigma_{j,x} = \sum_1^n \sigma_{i,j} = \sigma_{1,j} + \sigma_{2,j} + ... + \sigma_j^2 + \sigma_{n-1,j} + \sigma_{n,j}$$

Then Covariance $(X_j, X) = \sigma_j^2 + \sigma_{j,-j}$

And Variance $(X) = \sigma_x^2 = \sigma_j^2 + \sigma_{-j}^2 + 2\sigma_{j,-j}$

Since $\rho_{j,-j} = \dfrac{\sigma_{j,-j}}{\sigma_j \sigma_{-j}}$ then

Variance $(X) = \sigma_x^2 = \sigma_j^2 + \sigma_{-j}^2 + 2\rho_{j,-j}\sigma_j\sigma_{-j}$

replacing $\rho_{j,-j}$ by $\dfrac{\sigma_{j,-j}}{\sigma_j\sigma_{-j}}$,then

$$K_j = E(X_j / X > x_q) = \mu_j + \alpha\sigma_j{}^2(1 + \frac{\sigma_{j,-j}}{\sigma_j\sigma_{-j}}\frac{\sigma_{-j}}{\sigma_j})$$

$$K_j = \mu_j + \alpha(\sigma_j^2 + \sigma_{j,-j})$$

replacing $\sigma_j^2 + \sigma_{j,-j}$ by $\sigma_{j,x}$, then

$$K_j = \mu_j + \alpha\sigma_{j,x}$$

The allocation formula of sum of risks (X) is

$$K = \mu + \alpha\sigma_x^2$$

from the last two equations

$$K_j - \mu_j = (K - \mu)\frac{\sigma_{j,x}}{\sigma_x^2}$$

By letting $\beta_j = \dfrac{\sigma_{j,x}}{\sigma_x^2}$ then $K_j - \mu_j = \beta_j(K - \mu)$

7. Important Notes on Multivariate Normal Risks

Table B shows the means, standard deviations and correlation coefficients for 5 risks , each following the normal distribution. One of the correlation coefficients must be at least greater than - 0.25.

If all correlations are equality (- 0.25) except one (grater than - 0.25), then the total combined capital allocated by equality for risks of this coefficient and it will be zero for reminder of risks as shown in the following table.

Case	Total capital after the combining	capital allocated on risk 1	capital allocated on risk 2	capital allocated on risk 3	capital allocated on risk 4	capital allocated on risk 5
1	0.377	0.188	0.188	0.000	0.000	0.000
2	0.533	0.267	0.267	0.000	0.000	0.000
3	1.885	0.942	0.942	0.000	0.000	0.000
4	3.264	1.632	1.632	0.000	0.000	0.000
5	4.197	2.099	2.099	0.000	0.000	0.000
6	3.769	1.885	1.885	0.000	0.000	0.000
7	6.528	3.264	3.264	0.000	0.000	0.000

If all correlations are equality except one, total combined capital allocated by equality for risks of this coefficient and it will be equality for reminder of risks. As shown in the following table.

Case	Total capital after the combining	capital allocated on risk 1	capital allocated on risk 2	capital allocated on risk 3	capital allocated on risk 4	capital allocated on risk 5
1	0.377	0.188	0.188	0.000	0.000	0.000
2	0.533	0.267	0.267	0.000	0.000	0.000
3	1.885	0.942	0.942	0.000	0.000	0.000
4	3.264	1.632	1.632	0.000	0.000	0.000
5	4.197	2.099	2.099	0.000	0.000	0.000
6	3.769	1.885	1.885	0.000	0.000	0.000
7	6.528	3.264	3.264	0.000	0.000	0.000
8	4.943	1.782	1.782	0.460	0.460	0.460
9	7.270	3.166	3.166	0.313	0.313	0.313
10	3.286	0.605	0.605	0.692	0.692	0.692
11	1.643	0.303	0.303	0.346	0.346	0.346
12	9.793	4.896	4.896	0.000	0.000	0.000
13	0.979	0.490	0.490	0.000	0.000	0.000

When one correlation coefficient increases and the other correlation coefficients are fixed (do not change), the total capital combined increases. As shown in the following table.

Case	Total capital after the combining	ρ (1,2)	ρ (1,3)	ρ (1,4)	P (1,5)	ρ (2,3)	ρ (2,4)	ρ (2,5)	ρ (3,4)	ρ (3,5)	ρ (4,5)
1	0.377	-0.24	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
2	0.533	-0.23	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
3	1.885	0.00	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
4	3.264	0.50	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
5	4.197	0.99	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
6	3.769	0.00	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
7	6.528	0.50	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
8	4.943	0.00	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23
9	7.270	0.50	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23
10	3.286	-0.24	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23
13	0.979	0.5	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
16	17.879	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

If all correlation coefficients are fixed (grater than - 0.25) and the standard deviations are duplicated, then total capital combined is duplicated. Compare cases (3,6) , (4,7) , (10,11) , (4,12) and (12,13) in the following table..

Case	Total capital after the combining	σ1	σ2	σ3	σ4	σ5
3	1.885	1	1	1	1	1
4	3.264	1	1	1	1	1
6	3.769	2	2	2	2	2
7	6.528	2	2	2	2	2
10	3.286	2	2	2	2	2
11	1.643	1	1	1	1	1
12	9.793	3	3	3	3	3
13	0.979	0.3	0.3	0.3	0.3	0.3

If all correlations coefficients are equality (grater than - 0.25), then total capital combined is allocated for all risks by equality. As shown in the following table.

Case	Total capital after the combining	capital allocated on risk 1	capital allocated on risk 2	capital allocated on risk 3	capital allocated on risk 4	capital allocated on risk 5
14	5.960	1.192	1.192	1.192	1.192	1.192
15	11.919	2.384	2.384	2.384	2.384	2.384
16	17.879	3.576	3.576	3.576	3.576	3.576
17	10.322	2.064	2.064	2.064	2.064	2.064
18	20.645	4.129	4.129	4.129	4.129	4.129
19	8.428	1.686	1.686	1.686	1.686	1.686
20	1.192	0.238	0.238	0.238	0.238	0.238

The comparison for total capital before and after combining is shown in the following table.

Case	Total capital before the combining	Total capital after the combining
1	13.326	0.377
2	13.326	0.533
3	13.326	1.885
4	13.326	3.264
5	13.326	4.197
6	26.652	3.769
7	26.652	6.528
8	26.652	4.943
9	26.652	7.270
10	26.652	3.286
11	13.326	1.643
12	39.978	9.793
13	3.998	0.979
14	13.326	5.960
15	26.652	11.919
16	39.978	17.879
17	13.326	10.322
18	26.652	20.645
19	13.326	8.428
20	13.326	1.192

Case	Parameters of Two risks					The capital before combination			The capital after combination		
	μ 1	σ 1	μ 2	σ 2	ρ	K1	K2	K	K	K1	K2
1	4	2	4	2	0.000	9.330	9.330	18.661	15.538	7.769	7.769
2	2	3	2	3	0.500	9.996	9.996	19.991	17.849	8.924	8.924
3	3	5	3	5	1.000	16.326	16.326	32.652	32.652	16.326	16.326
4	0	0.5	0	0.5	-0.500	1.333	1.333	2.665	1.333	0.666	0.666
5	5	2	5	4	-1.000	10.330	15.661	25.991	15.330	-0.330	15.661
6	1	0.9	1	0.9	0.500	3.399	3.399	6.797	6.155	3.077	3.077
7	10	5	10	5	0.500	23.326	23.326	46.652	43.081	21.541	21.541
8	20	2	20	4	0.500	25.330	30.661	55.991	54.103	24.029	30.074
9	8	1	8	1	-0.500	10.665	10.665	21.330	18.665	9.333	9.333
10	9	1	9	4	-0.500	11.665	19.661	31.326	27.610	8.261	19.349
11	11	2	11	4	-0.500	16.330	21.661	37.991	31.233	11.000	20.233
12	e	2	e	8	-0.250	8.049	24.040	32.089	26.081	2.718	23.363
13	30	1	30	1	0.250	32.665	32.665	65.330	64.214	32.107	32.107
14	1	3	1	6	-0.750	8.996	16.991	25.987	13.308	-1.827	15.134
15	7	0.25	7	1	0.750	7.666	9.665	17.332	17.195	7.556	9.640
16	4	2	4	2	1.000	9.330	9.330	18.661	18.661	9.330	9.330
17	2	3	2	4	1.000	9.996	12.661	22.656	22.656	9.996	12.661
18	3	5	2	5	1.000	16.326	15.326	31.652	31.652	16.326	15.326
19	1	6	2	7	1.000	16.991	20.656	37.648	37.648	16.991	20.656
20	4	3	5	6	-0.750	11.996	20.991	32.987	20.308	1.173	19.134
21	2	3	1	6	-0.750	9.996	16.991	26.987	14.308	-0.827	15.134
22	6	1	6	2	-1.000	8.665	11.330	19.996	14.665	3.335	11.330
23	2	6	2	7	-0.900	17.991	20.656	38.648	12.171	0.435	11.736

Table A

Case	The means					The standard deviations					quantile	The correlations coefficients									
	μ_1	μ_2	μ_3	μ_4	μ_5	σ_1	σ_2	σ_3	σ_4	σ_5	q	$\rho_{(1,2)}$	$\rho_{(1,3)}$	$\rho_{(1,4)}$	$\rho_{(1,5)}$	$\rho_{(2,3)}$	$\rho_{(2,4)}$	$\rho_{(2,5)}$	$\rho_{(3,4)}$	$\rho_{(3,5)}$	$\rho_{(4,5)}$
1	0	0	0	0	0	1	1	1	1	1	0.99	-0.24	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
2	0	0	0	0	0	1	1	1	1	1	0.99	-0.23	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
3	0	0	0	0	0	1	1	1	1	1	0.99	0	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
4	0	0	0	0	0	1	1	1	1	1	0.99	0.5	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
5	0	0	0	0	0	1	1	1	1	1	0.99	0.99	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
6	0	0	0	0	0	2	2	2	2	2	0.99	0	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
7	0	0	0	0	0	2	2	2	2	2	0.99	0.5	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
8	0	0	0	0	0	2	2	2	2	2	0.99	0	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23
9	0	0	0	0	0	2	2	2	2	2	0.99	0.5	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23
10	0	0	0	0	0	2	2	2	2	2	0.99	-0.24	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23
11	0	0	0	0	0	1	1	1	1	1	0.99	-0.24	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23
12	0	0	0	0	0	3	3	3	3	3	0.99	0.5	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
13	0	0	0	0	0	0.3	0.3	0.3	0.3	0.3	0.99	0.5	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25
14	0	0	0	0	0	1	1	1	1	1	0.99	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	2	2	2	2	2	0.99	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	3	3	3	3	3	0.99	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	1	1	1	1	1	0.99	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
18	0	0	0	0	0	2	2	2	2	2	0.99	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
19	0	0	0	0	0	1	1	1	1	1	0.99	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
20	0	0	0	0	0	1	1	1	1	1	0.99	-0.24	-0.24	-0.24	-0.24	-0.24	-0.24	-0.24	-0.24	-0.24	-0.24

Table B

8. Observations and Conclusions

The key result of this paper is that in the case of the multivariate normal distribution. Tail VaR is one of many possible coherent risk measures however, the tail VaR based allocation method of the capital for combined risk units works on:

1- Reduction of capital allocated of each risk for bivariate risks or multivariate risks.
2- Determination of percentage allocation of total capital to each risk unit of business.
3- The allocation methodology results can be applied to any financial units.

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توزيع وتخفيض رأس المال المخصص لكل خطر بعد دمج الوحدات المالية

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ملخص :

يتناول هذا البحث دراسة تأثر رأس المال الكلى بدمج الوحدات المالية التى يتبع أخطارها التوزيع الطبيعى المتعدد المتغيرات. كما يهدف البحث إلى مدى إمكانية تخفيض رأس المال الكلى ورأس المال المطلوب لكل خطر بعد دمج جميع الأخطار وكذلك تحديد النسب المئوية للتوزيع بين الوحدات المدمجة. نتائج منهجية هذا التوزيع يمكن تطبيقها على بعض العناصر والمجموعات المالية المتكاملة كشركات مالية أو شركات تأمين ، أوشركات غير مالية لتحديد المخزون السلعى لكل شركة بعد دمج تلك الشركات.